A GALERKIN FORMULATION OF THE GENERALIZED HELMHOLTZ DECOMPOSITION FOR VORTICITY METHODS

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Vorticity formulations for the incompressible Navier-Stokes equations have certain advantages over primitive-variable formulations including a reduction in the number of equations to be solved through the elimination of the pressure variable, identical satisfaction of the compressibility constraint and the continuity equation, an implicitly higher-order approximation of the velocity components, and a reduced computational domain. However, the accurate implementation of the boundary conditions seems to continue to be an impediment to the accpetance and use of numerical methods based on vorticity formulations. Velocity boundary conditions can be implicitly satisfied by maintaining the kinematic compatibility of the velocity and vorticity fields as described by the generalized Helmholtz decomposition (GHD). The GHD is a classical mathematical decomposition of a vector field into its rotational and irrotational parts. Previous numerical implementations of the GHD have been based on point collocation. Point collocation approaches can lead to poorly approximated velocity boundary conditions. Further, because of the inherant errors associated with point collocation methods, previous investigators have had to impose a variety of ad hoc integral contraints on vorticity formulations in order to stabilize their numerical methods In the current research, a Galerkin implementation of the GHD is shown to provide solutions for the boundary vortex sheet strengths that result in satisfying the velocity boundary conditions orders of magnitude better than point collocation implementations. To show the power of the current approach, a variety of interior and exterior flows are considered including flow over a backwards facing step, driven cavity flow, and wake flows about blunt bodies. Because of the efficiencies of the vorticity method, hundreds of body diameters can be included in the wake flow analyses enabling the study of wake transitions. Finally, natural convection in an 8x1 enclosure is considered. This problem has not typically been studied using vorticity methods. However, the natural convection problem clearly demonstrates the capability of the vorticity formulation to admit additional advection-diffusion equations such as would be required to study certain turbulence models or chemically-reacting flows.